TECHNICAL NOTE

D-954

A METHOD FOR LONGITUDINAL AND LATERAL RANGE CONTROL

FOR A HIGH-DRAG LOW-LIFT VEHICLE ENTERING THE

ATMOSPHERE OF A ROTATING EARTH

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON   September 1961

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SUMMARY

A study has been made of a method for controlling the trajectory of a high-drag low-lift entry vehicle to a desired longitude and latitude on the surface of a rotating earth. By use of this control technique the vehicle can be guided to the desired point when the present position and heading of the vehicle are known and the desired longitude and latitude are specified. The present study makes use of a single reference trajectory and an estimate of the lift and side-force capabilities of the vehicle. This information is stored in a control-logic system and used with linear control equations to guide the vehicle to the desired destination. Results are presented of a number of trajectory studies which describe the operation of the control system and illustrate its ability to control the vehicle trajectory to the desired landing area.

INTRODUCTION

In order to assure the recovery of an entry vehicle, it is desirable for the vehicle to arrive at a predetermined area on or above the earth's surface. Thus, the vehicle must be controlled in the atmosphere in such a manner that its trajectory terminates at the desired destination. For a vehicle with lifting capabilities, this control can be achieved by regulating the lift force so that the vehicle will simultaneously traverse the desired range and obtain the desired heading.

The problem of control of an entry vehicle to a desired landing area has been studied extensively. (See, for example, refs. 1 and 2.) These studies, in general, have provided only for control of the vehicle's range along the flight path (longitudinal range). However, under practical conditions lateral or cross-range control of the vehicle may be as important in arriving at the desired destination as control of the longitudinal range.

The present paper describes a control technique capable of guiding
an entry vehicle to a desired point when the present position and heading
of the vehicle are known and the desired destination specified in longi-
tude and latitude. In addition to present position and heading, a single
reference trajectory and an estimate of the lift and side-force capabili-
ties of the vehicle are stored in a control-logic system. The manner in
which these quantities can be obtained is discussed and a control-logic
scheme using the stored information along with linear control equations
is described. In the study, preliminary runs were made on a digital
computer to establish the method and control-equation gains. Then a
large number of runs were made on the digital computer to verify the
operation of the range-guidance system. The results of a number of
these trajectory studies illustrating the problems and procedures are
given.

SYMBOLS

In the present paper, distances are measured in the U.S. foot.
(One U.S. foot = 0.3048006 meter.)

A	orbital heading (fig. 1), deg
a	acceleration due to aerodynamic forces, ft/sec^2
b,c	constants used in control-logic system (figs. 4 and 5)
C_c	chord-force coefficient
C_N	normal-force coefficient
C_Y	side-force coefficient
F	aerodynamic force, lb
g	acceleration due to gravity, ft/sec^2
H	angle between X_i-axis and X_e-axis ($\dot{H} = \omega_e$), radians
h	altitude above surface of earth, ft
$\bar{i},\bar{j},\bar{k}$	unit vectors along earth-stabilized axes
K_1,K_2,K_3	gain constants used in control equations
L/D	lift-drag ratio

L_c	colatitude, deg
q	dynamic pressure, lb/sq ft
R	radial distance to vehicle measured from earth's geographic center, ft
R_e	radius of earth, ft
r	range-to-go (great-circle distance from vehicle's present position to desired destination), miles
S	surface area, sq ft
u,v,w	component of velocity along X-, Y-, and Z-axis, respectively, ft/sec
V	velocity, ft/sec
W	weight of vehicle, lb
X,Y,Z	earth-stabilized axes (origin at center of gravity of body); Z-axis is positive toward earth's center and X-axis is positive toward south
X_b,Y_b,Z_b	body axes
X_e,Y_e,Z_e	polar earth axes (fixed in earth); Z_e-axis is positive toward north
X_i,Y_i,Z_i	polar inertial axes (fixed in space); Z_i-axis is positive toward north
X_w,Y_w,Z_w	wind axes
α	angle of attack, deg
α_1	computed angle of attack from control equations, deg
α_2	estimated angle of attack from control logic, deg
β	angle of sideslip in body axes system, deg
β_1	computed angle of sideslip from control equations, deg
γ	flight-path angle, deg

ϵ_A error in orbital heading, deg

ϵ_r longitudinal range-to-go error, miles

η inertial longitude, $\lambda + H$, deg

λ longitude, deg

ψ, θ, ϕ Euler angles used to orient vehicle axes with respect to local earth axes, radians

ω_e angular velocity of earth, radians/sec

Subscripts:

D   desired destination

i   inertial

o   value of variable at zero time

Dots above quantities denote differentiation with respect to time.

A bar over a symbol indicates a vector.

ANALYSIS

The assumptions and procedures used in developing a method for guiding a low-lift-drag-ratio entry vehicle to a predetermined destination on the earth are described in the following sections.

Vehicle Characteristics

The vehicle was assumed to be of the high-drag low-lift type of entry vehicle with a wing loading of 20 pounds per square foot. The assumption was made that the vehicle had total lifting capabilities corresponding to a lift-drag ratio of about 0.5. Thus, if lift is employed simultaneously for longitudinal control and lateral maneuvering, the vector sum of the components cannot exceed 0.5 of the drag.

Initial Conditions

Initial conditions for the study were established by assuming that the vehicle had completed a retromaneuver and was entering the earth's atmosphere with an inertial velocity of 25,710 feet per second at an altitude of 350,000 feet. Various orbital headings and initial conditions on latitude and longitude were assumed. In most entries the initial flight-path angle of the vehicle was assumed to be -1°. However, the effect of entries with initial flight-path angles of -0.5° and -1.5° were also considered. All entries were terminated at an altitude of 100,000 feet since at this altitude the vehicle was descending almost vertically and very little range capability remained.

Equations of Motion

The geometry used in defining the position of an entry vehicle with respect to a rotating earth is given in figure 1. The force equations solved in this analysis were derived in reference 1 and are as follows:

$$
\left.
\begin{aligned}
\ddot{L}_c &= \frac{a_X}{R} - \frac{2\dot{h}\dot{L}_c}{R} + \dot{\eta}^2 \cos L_c \sin L_c \\[2em]
\ddot{\eta} &= -\frac{a_Y}{R \sin L_c} - \frac{2\dot{h}\dot{\eta}}{R} - 2\dot{L}_c\dot{\eta}\,\frac{\cos L_c}{\sin L_c} \\[2em]
\ddot{h} &= -a_Z + R\dot{L}_c^2 + R\dot{\eta}^2\sin^2 L_c - g\,\frac{R_e^2}{R^2} \\[2em]
\dot{\lambda} &= \dot{\eta} - \omega_e
\end{aligned}
\right\}
\qquad (1)
$$

The body axes were related to the earth-stabilized axes by the usual Euler angle sequence ψ, θ, ϕ. For simplicity the roll angle ϕ was held at zero since the vertical and side forces could be attained by combinations of angle of attack and angle of sideslip. The same effect could have been achieved by using angle of attack and roll angle with $\beta = 0$. From the standpoint of mathematical analysis, either method can be used. The yaw angle ψ and pitch angle θ are as follows:

$$\left.\begin{array}{l} \psi = A - \beta \\ \\ \theta = \gamma + \alpha \end{array}\right\} \tag{2}$$

where

$$\left.\begin{array}{l} A = \tan^{-1} \dfrac{v}{u} \\ \\ \gamma = \sin^{-1}\left(- \dfrac{w}{V}\right) \\ \\ \overline{V} = u\overline{i} + v\overline{j} + w\overline{k} = R\dot{L}_c\overline{i} - R\dot{A} \sin L_c\overline{j} - \dot{h}\overline{k} \end{array}\right\} \tag{3}$$

The vehicle aerodynamic characteristics were assumed to be the same as for a slightly rounded plate $\left(\dfrac{\text{radius}}{\text{chord}} \approx 3.15\right)$ normal to the airstream. The aerodynamic forces in the body axes are given by

$$\left.\begin{array}{l} F_{Xb} = C_N q S \\ \\ F_{Yb} = C_Y q S \\ \\ F_{Zb} = C_c q S \end{array}\right\} \tag{4}$$

The aerodynamic force coefficients were assumed to vary in the following manner:

$$C_N = -1.7 \cos^2\alpha \, \cos^c\beta$$

$$C_Y = -0.1 \sin \beta$$

$$C_c = -0.1 \sin \alpha$$

It should be noted that an angle of attack of zero refers to the condition in which the surface is normal to the flow. The vehicle obtains lift in the plane of flight by varying its angle of attack about this normal position (for positive lift $\alpha < 0^\circ$). In order to obtain control forces lateral to the plane of flight, the vehicle must vary the sideslip angle β.

The forces of equations (4) are transformed by Euler angle conversions and divided by the mass to give a_X, a_Y, and a_Z.

For this analysis it was assumed that the earth was spherical with a radius of 3,963 miles. The atmosphere was assumed to rotate with the earth, and no relative movement (wind shear) was assumed. The atmospheric density was assumed to vary in accordance with the 1959 ARDC model atmosphere (ref. 3).

Range Capability

The longitudinal and lateral range capabilities of the vehicle for different initial orbital headings are shown in figure 2 for a particular set of initial conditions. These "footprints" represent the locus of the end points of trajectories for which sideslip angle and angle of attack were adjusted so that the maximum range capability of the vehicle was obtained with the assumption that these angles be held constant for any particular trajectory at values which produce an L/D of 0.5.

Figure 2(a) shows the maximum range capability for equatorial entries for which the initial orbital headings were toward the east and west. This figure shows that the range capability is greater for entries toward the west than for entries toward the east. This increase occurs because the velocity is greater relative to the atmosphere for entries toward the west and hence the dynamic pressure acting on the vehicle is greater. Figure 2(b) shows the maximum range capability for entries initiated at the equator and with initial orbital headings toward the north and south. Again, the land area attainable is larger for these entries than for entries toward the east because of the higher velocity with respect to the atmosphere and because of the earth's rotation which increases the vehicle's lateral range in the direction opposite to the rotation.

These footprints are used as a basis for comparing the maximum range capability of the vehicle with the range obtainable by using a range-control system.

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Guidance Geometry

The geometry used to establish the vehicle position with respect to the desired destination is shown in figure 3. The great circle distance from the present position of the vehicle (λ, L_c) to the desired destination $(\lambda_D, L_{c,D})$ is defined as the range-to-go r and is represented in figure 3 by the curve from point O to point D. By use of spherical trigonometry this range-to-go can be calculated by the following equation:

$$r = \cos^{-1}\left[\cos L_c \cos L_{c,D} + \sin L_c \sin L_{c,D} \cos(\lambda - \lambda_D)\right] \qquad (5)$$

The vehicle orbital heading A was defined in the previous section by the first of equations (3). The vehicle desired orbital heading A_D is given by the following equation:

$$A_D = \sin^{-1} \frac{\sin(\lambda_D - \lambda)\sin L_{c,D}}{\sin r} - 180° \qquad (6)$$

The difference in these headings $A_D - A$ gives an error in heading ϵ_A as shown in figure 3. The altitude, range-to-go, and error in heading were used in the guidance scheme to determine the manner in which the lift force of the vehicle should be used.

Guidance Technique

Range control along the flight path of the vehicle (longitudinal range) was achieved by controlling the vehicle trajectory to a reference trajectory of altitude as a function of range-to-go which terminated at the desired destination. This reference trajectory gives desired values for range-to-go as a function of altitude. By continually comparing the vehicle range-to-go with the reference range-to-go at the present altitude, an error signal is obtained. This error signal coupled with the rate of change of the error signal is then used to command an angle of attack different from that of the reference trajectory; this α increases or decreases the vehicle lift and thereby the trajectory approaches the reference trajectory. The equation for the command angle of attack is of the form:

$$\alpha_1 = \alpha_T + K_1 \epsilon_r + K_2 \dot{\epsilon}_r \qquad (7)$$

where α_T is the angle of attack used in computing the reference trajectory $(\alpha_T = -12°)$.

The lateral range of the vehicle was controlled by using the vehicle heading error as an error signal to control the sideslip angle in such a manner that the heading error would be reduced to zero. The equation for the sideslip angle is

$$\beta_1 = K_3 r \sin \epsilon_A \tag{8}$$

The nominal gains K_1, K_2, and K_3 used in the control equations are as follows:

$$K_1 = 2° \text{ per mile}$$

$$K_2 = 4° \text{ per mile per sec}$$

$$K_3 = 4° \text{ per mile}$$

These gains are such that α_1 and β_1 were maintained at their maximum allowable limits throughout most of an entry, which was necessary in order to keep the terminal range errors small. The limiting values for α_1 and β_1 and their determination are discussed in the section entitled "Control Logic." The effect of using smaller gains is discussed in the section "Results and Discussion."

Control Logic

The L/D restrictions and high gains discussed previously made it necessary to employ a control-logic scheme which proportioned the amount of lift available for control in the longitudinal and lateral directions as a function of the range errors in each direction. This proportioning was done in order to maintain an adequate component of lift for control in each direction without sacrificing needed lift. In other words, this selection process is necessary to prevent wasteful use of the vehicle's lifting capability in one plane of flight which would result in a decrease in range capability in the other plane.

The purpose of the control-logic system is to limit angle of attack as a function of the initial range-to-go and as a function of the heading

errors. This limiting of angle of attack is done by estimating the
angle of attack which, if held constant throughout the trajectory,
would cause the vehicle to approximately traverse the desired longitu-
dinal range. By allowing the vehicle angle of attack to vary within
a range of values about this estimated value and by using the remainder
of the vehicle's lifting capability for lateral control, an efficient
use is made of the vehicle's total lift. This estimation is achieved
by assuming that the vehicle is traveling over a nonrotating earth and
that there are no changes in heading during the entry. This assumption
greatly simplified the estimation since longitudinal range is a function
of lateral range and orbital heading. A block diagram of the control-
logic system, shown in figure 4, is explained with the aid of the curves
in figure 5.

With the previous assumptions a constant angle of attack α_2 can
be computed as a function of initial range-to-go which will cause the
vehicle to approximately traverse the desired range. The variation in
α_2 with initial range-to-go is shown in figure 5(a). The control logic
then selects limits on angle of attack which depend on the magnitude of
the estimated α_2. These limits allow the angle of attack to vary
slightly about the value for α_2 as shown in figure 5(a) (between $\alpha_2 + b$
and $\alpha_2 - c$). This variation is necessary to correct for small changes
in range due to changes in heading, the effect of the earth's rotation,
and any other perturbing influences on the vehicle during entry. A
straight-line variation in b and c with α_2 was used in the study.
This variation is shown in figure 5(b). Since the addition of b to α_2
increases the lift in the plane of flight but decreases the lateral lift,
a variation in b was chosen, which would not greatly restrict the
available sideslip angle throughout the angle-of-attack range. A larger
variation was allowed in c than in b since the lower limits on α
had less effect on the vehicle longitudinal and lateral ranges. This
wider variation was also necessary to overcome a tendency by the vehicle
to overshoot the desired area for certain headings. It should be noted
that for a particular entry the values for α_2, b, and c are fixed
by the initial range-to-go and remain constant throughout the entry.

After the allowable range of angle of attack has been established,
the actual value used at any point is obtained by comparing the value
calculated by equation (7) α_1 with the estimated value α_2 and
selecting a value which is between the limits specified in figure 5(a).

After the angle of attack has been established, the remainder of
the vehicle's lifting capability can be employed for lateral control if
needed. Hence the control logic (fig. 4) computes a value for the side-
slip angle β which produces a lift component lateral to the plane of
flight so that, when it is combined geometrically with the lift component

in the plane of flight, a total lift-drag ratio of about 0.5 results $\left(\sqrt{\alpha^2 + \beta^2} = 30^\circ\right)$. This value for β is then compared with the value for β_1 computed by use of equation (8) to determine if this much lifting capability is required in the lateral direction. If the total remaining lifting capability is not required for lateral control (ϵ_A is small or zero), then the control logic readjusts the limits on α_1 to allow a wider variation and hence more exact control of longitudinal range. This procedure is repeated continuously throughout the entry trajectory and gives closed-loop control over angle of attack and side-slip angle.

RESULTS AND DISCUSSION

Reference Trajectory

The reference trajectory of range-to-go as a function of altitude used in this analysis was computed for a nonrotating earth with constant values of angle of attack and sideslip angle held throughout the trajectory. The particular reference trajectory used was computed with an angle of attack of -12° and a sideslip angle of 26°. These values correspond to a condition for which the lift-drag ratio is about 0.5. Although only longitudinal range was controlled by using this reference trajectory, it was found that better control during lateral maneuvers was obtained by using a reference trajectory computed with a near-maximum sideslip angle.

Operation of Control System

In order to illustrate the operation of the control system, typical trajectories were calculated for entries with a large initial heading error and with no initial heading error. The variations in some of the trajectory variables with range for these entries is shown in figure 6. In both equatorial entries the initial heading was toward the east; the desired longitudinal range was 2,500 miles. However, for one entry no lateral range was called for, whereas for the other entry a lateral range of 200 miles was desired.

For the entry with no heading error the sideslip angle was zero so that the vehicle's full lifting capability was available for controlling the longitudinal range. Therefore, the angle of attack was allowed to vary between the maximum limits (± 30) when required. Figure 6 shows several angle-of-attack oscillations between the maximum limits, and, as the error in range-to-go and the rate of change of this error

approached zero near the end of the trajectory, the command angle of attack approached the reference value. For this entry, the final range error at an altitude of 100,000 feet, was 5 miles. It is possible that the gains in the system were too large as the angle-of-attack oscillations persisted in a manner which appeared only lightly damped. It was found, however, that these oscillations did not affect to any great extent the final range error. Although a variable-gain system might have produced superior results, for simplicity constant gains were used.

For the entry with a desired lateral range of 200 miles, figure 6 shows that the control logic selected a maximum angle of attack of about -22.5° and the remainder of the lifting capability was used to control lateral range (β = -20°). This condition remained until the range-to-go was 1,130 miles at which time the error in range-to-go had been reduced to zero (vehicle crossed the reference trajectory). At this point the control logic commanded less angle of attack (α = -5°) which allowed the sideslip angle to be increased to -29.5°. This oscillation in α and β continued until the longitudinal and lateral errors were reduced to zero near the end of the trajectory. For this entry the final errors in longitudinal and lateral ranges were each less than 1 mile at an altitude of 100,000 feet above the desired destination. The smaller terminal errors for this entry, as compared with those for the entry with no desired lateral range, may have resulted from the fact that the reference trajectory was computed with a sideslip angle applied and hence represented a more optimum condition for the entry with an initial lateral error or from the fact that the lower angle-of-attack limits reduced the tendency of the system toward instability.

It is seen in figure 6 that, as the heading error is reduced to zero, the sideslip angle also goes to zero and there is no tendency to overshoot the desired heading. Hence, no damping was required in the side-force equation. Also of interest is the fact that deceleration remained below 3g for both entries even though negative lift was used for considerable periods for the entry with no heading error. Figure 6 also shows altitude as a function of range-to-go for the reference trajectory to which the vehicle's longitudinal range was controlled. This same reference trajectory was used for all entries regardless of the initial heading or the initial range errors.

Control-Equation Gains

It is seen in figure 6 that angle of attack and sideslip angle appear to be controlled in an on-off manner throughout most of the trajectory. (Actually the change from maximum positive lift to maximum negative lift occurred over a time interval of from 5 to 10 seconds.) This change resulted from the high gains used in the control equations (K_1, K_2, and K_3). Although no attempt was made to arrive at optimum

values for these gains, it was found that high gains were necessary in order to keep the terminal range errors small. For example, when the trajectories shown in figure 6 were computed with the gains on the control equations reduced by a factor of 4, the terminal errors in longitudinal and lateral range are increased by about a factor of 10.

Entries With Different Initial Headings

Numerous trajectories were computed for entries with different combinations of headings and desired longitudinal and lateral ranges. Since the variations in the trajectory variables for these entries were similar to those shown in figure 6, detailed histories for these trajectories are not presented. However, a comparison of some of the significant trajectory variables for similar entries with different initial headings and a general discussion of these entries are given.

A comparison of the error in range-to-go, heading error, and deceleration for typical entries to the east, west, and south is shown in figure 7. For each of these entries the desired longitudinal and lateral ranges were 1,800 and 100 miles, respectively. Since the initial range-to-go was the same for all of these entries, the values for a, α_2, b, and c were also the same.

It is seen that the error in range-to-go was reduced to zero sooner for the entry toward the east than for the other two entries. This condition occurred because the initial range-to-go was less than the reference range-to-go and, hence, the earth rotation helped to correct for this initial error. (The reference trajectory rotated toward the vehicle trajectory.) The final errors in the longitudinal and lateral ranges were each less than 2 miles for this entry toward the east. Note that the error in heading for the entry toward the south was corrected before the heading errors for the other entries. For this entry the earth's rotation aids in correcting the heading error. (The lateral error was toward the east.) The heading error for the entry toward the east was corrected quicker than that for the entry toward the west since the range-to-go error was corrected sooner and, hence, more lateral control force was available. The terminal errors for the entries toward the east and west were all less than 3 miles.

The maximum decelerations for the entries toward the west and south were greater than the maximum deceleration for the entry toward the east because of the higher initial velocities for these headings. For all three entries this peak deceleration was less than 4.5g. It was found that even under the most extreme conditions investigated (short desired ranges calling for negative lift during part of the entry) the maximum deceleration remained below 7g.

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For all entries calculated it was found that, since the reference trajectory moves with the earth, no corrections had to be made to the reference trajectory to compensate for the earth's rotation. For example, on an entry toward the east it was not necessary to aim initially for a point ahead of the desired landing area to correct for the movement of the landing area during entry. It was also found that for entries toward the north or south it was not necessary to aim ahead or behind the desired heading since the lateral correction system was adequate to compensate for the earth's rotation.

Range Capability for Controlled Vehicle

The maximum range capability for the vehicle was described previously and shown in figure 2. These maximum range contours were computed for ideal conditions (with constant angles of attack and sideslip producing a total L/D of 0.5); hence, the range capability for the controlled vehicle will be smaller since the angles of attack and sideslip were varied throughout the entry trajectory.

The range capability for the controlled vehicle was assumed to be that area inside which the vehicle could be controlled to within a radius of 10 miles of the desired destination at an altitude of 100,000 feet above the desired area. Numerous trajectories were calculated to establish this maximum controllable area. The results are shown in figure 8. In this figure a comparison is presented of the range capability using controls with the maximum range capability of the vehicle for entries with the initial heading toward the east. This figure shows that the vehicle's range can be controlled over about 90 percent of its maximum capability. Similar contours were established for entries toward the north and west with comparable results. Hence, it is possible to control the vehicle within an area which is only slightly less than the total range capability of the vehicle.

Entries With Initial Flight-Path Angles Different From -1°

In order to investigate the operation of the control system for entries with initial flight-path angles other than -1°, a series of trajectories were computed for initial flight-path angles of -0.5° and -1.5°. The reference trajectory and control logic used for these entries were the same as those previously used. The results of some of these entries are shown in figure 9. This figure presents lateral range, altitude, and deceleration as a function of range-to-go for entries with initial flight-path angles of -0.5° and -1.5° and desired longitudinal ranges of 2,500 and 1,800 miles, respectively. The desired lateral range for both entries was 100 miles.

It is seen that for both entries the vehicle was controlled to the desired destination with good accuracy. The terminal errors in both cases were less than 6 miles. Note that the deceleration was greater for the -0.5° entry than for the -1.5° entry since negative lift was required for most of the entry.

Additional entries calculated for initial entry angles of -0.5° and -1.5° indicated that the vehicle could be guided to the vicinity of the target over a large part of the vehicle's range capability for these entry angles. However, the accuracy and controllable area were reduced in comparison with those shown in figure 8; hence, the vehicle's controllable range capability would undoubtedly be increased if it were controlled to a reference trajectory which more closely matched the vehicle initial entry angle.

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SUMMARY OF RESULTS

Results of a study of a method for guidance of an entry vehicle to a desired point on the earth's surface may be summarized as follows:

1. The reference trajectory, heading-error method of reentry range control considered in this study provides longitudinal and lateral range control over about 90 percent of the vehicle's range capability for the entry for which the initial entry angle is similar to the reference-trajectory initial entry angle.

2. High gains in the control equations which resulted in maximum values for angle of attack and sideslip angle were necessary to keep the terminal range errors at a minimum (within a ±10-mile radius of the target).

3. It was found that the control procedure was sufficient to compensate for the earth's rotation.

4. One reference trajectory of range as a function of altitude was sufficient for range control regardless of the vehicle initial heading providing the desired destination is within the range capability of the vehicle.

5. Although negative lift was required during portions of the controlled trajectories, the peak decelerations remained within reasonable limits (usually less than 4g).

6. A reference trajectory based upon maximum lateral range was found to give better range control than one based on no lateral range.

Langley Research Center,
    National Aeronautics and Space Administration,
        Langley Field, Va., June 29, 1961.

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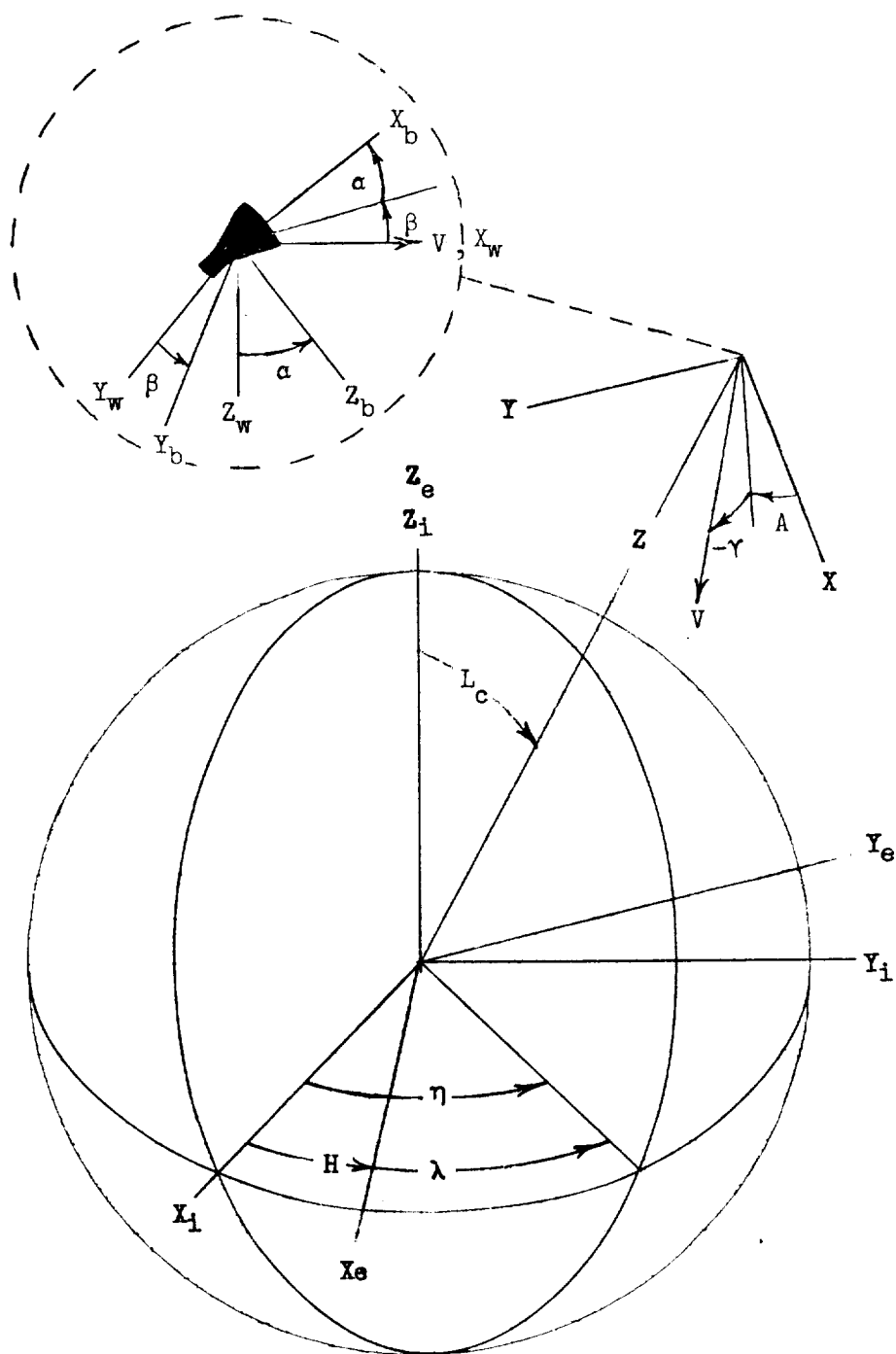
Figure 1.- Geometry of axes systems used in defining the position of an entry vehicle with respect to a rotating earth.

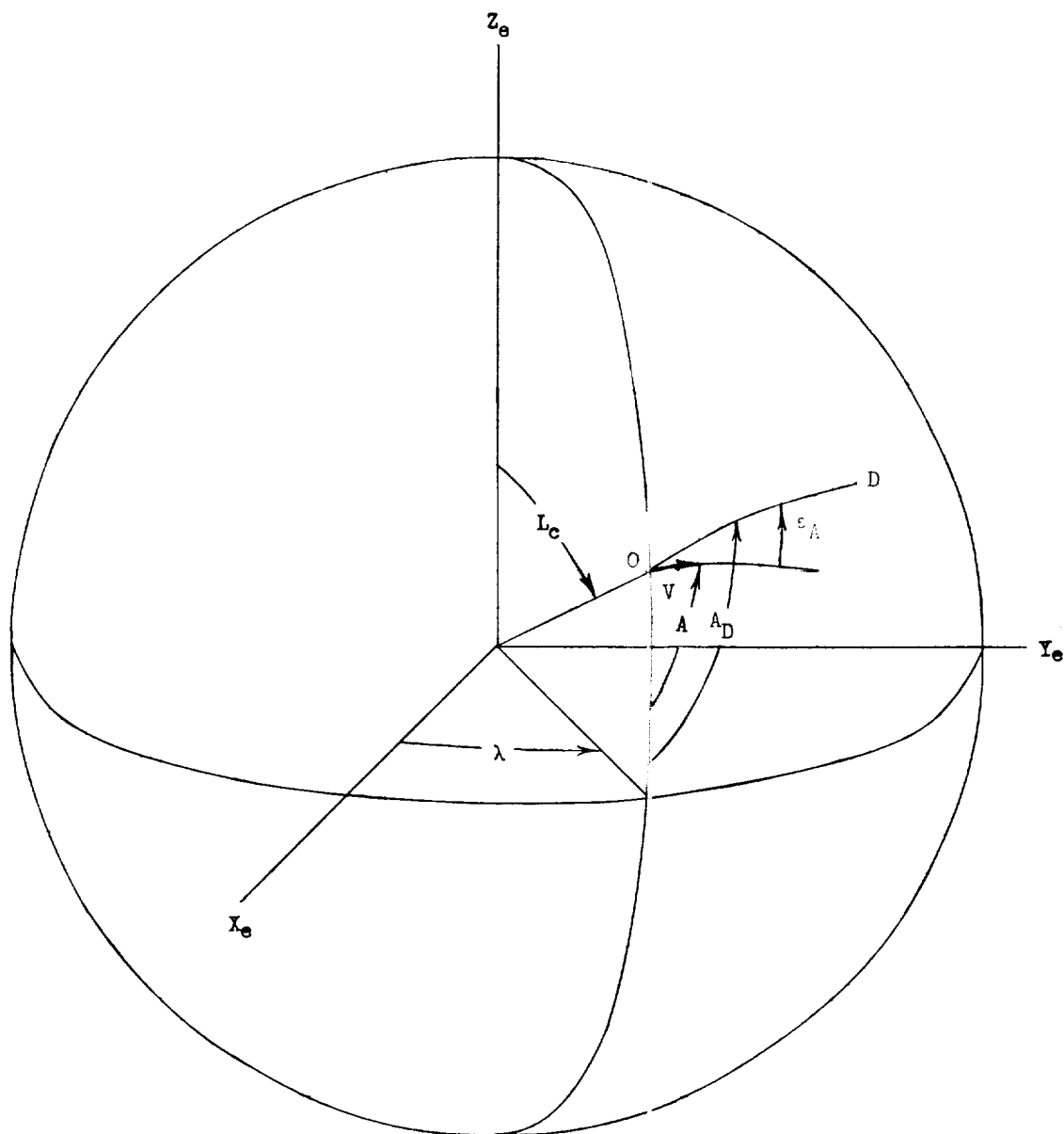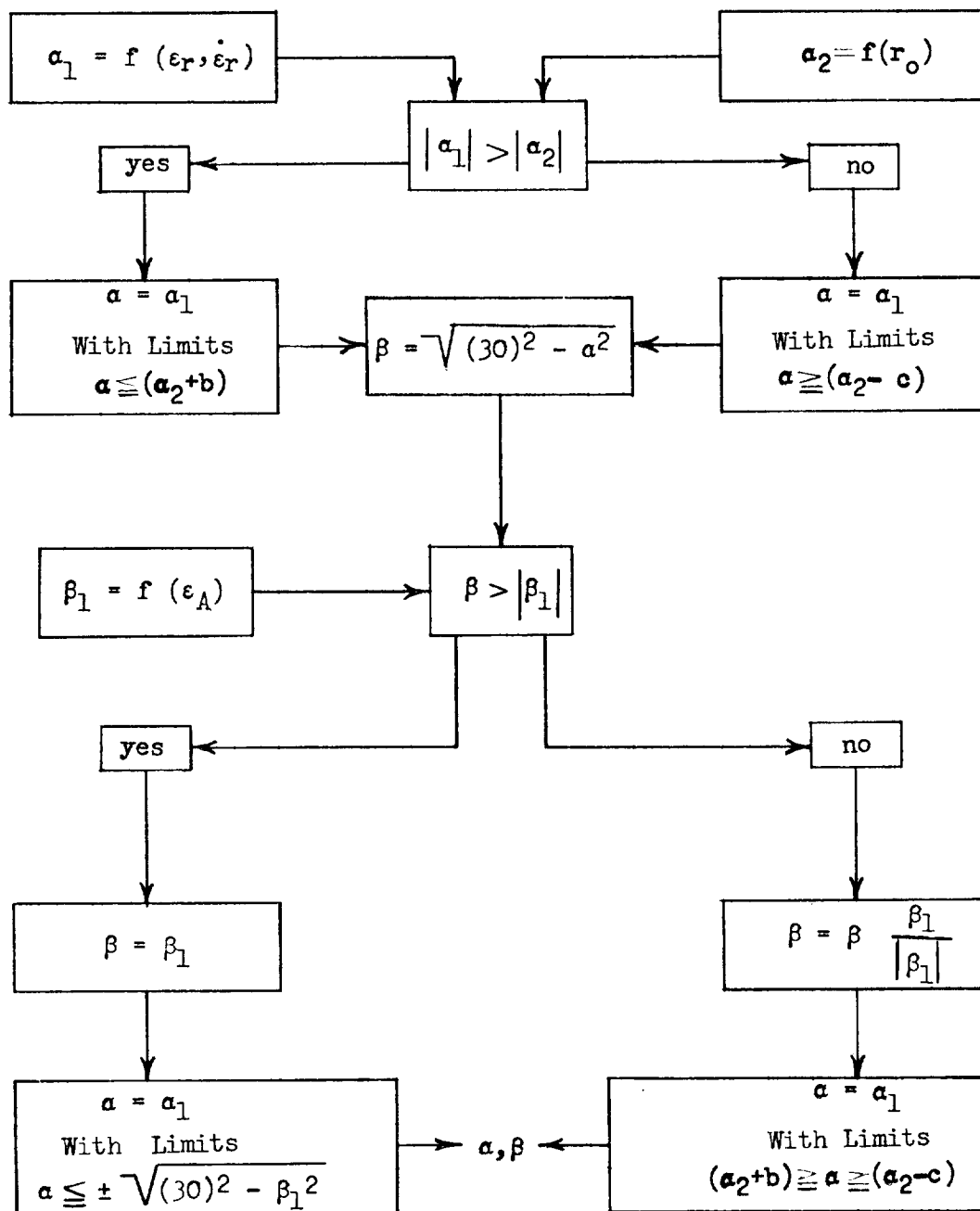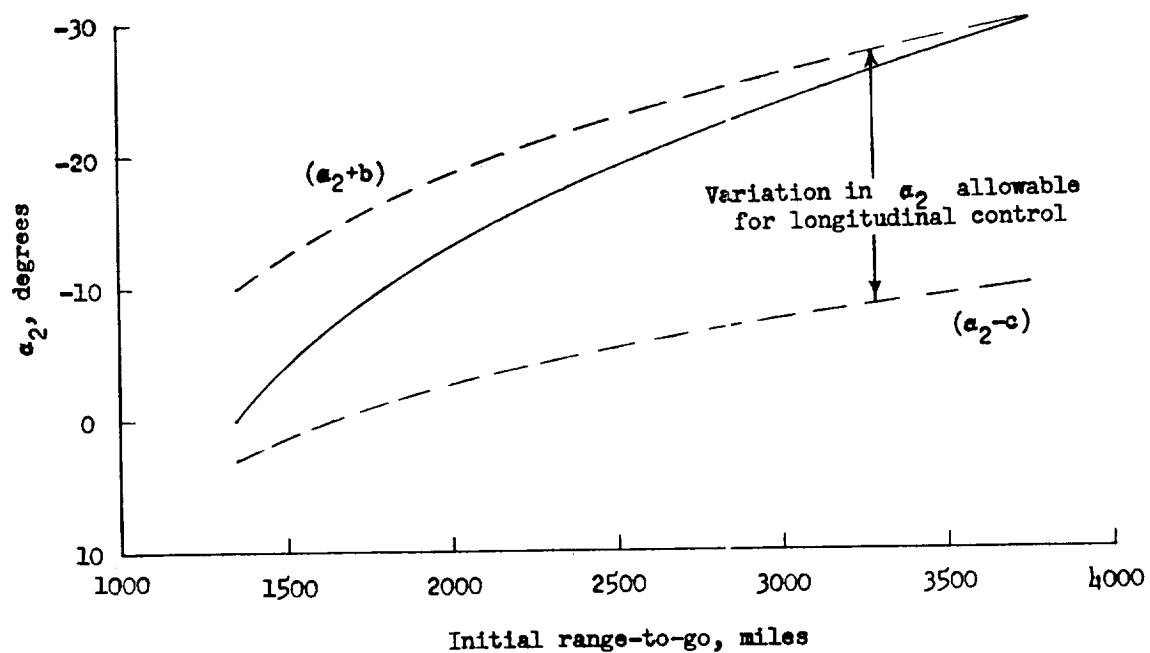Figure 3.- Guidance geometry used in longitudinal and lateral range
control.

L-1660

$\alpha_1 = f(\varepsilon_r, \dot{\varepsilon}_r)$

$\alpha_2 = f(r_o)$

$|\alpha_1| > |\alpha_2|$

yes

no

$\alpha = \alpha_1$
With Limits
$\alpha \leqq (\alpha_2 + b)$

$\beta = \sqrt{(30)^2 - \alpha^2}$

$\alpha = \alpha_1$
With Limits
$\alpha \geqq (\alpha_2 - c)$

$\beta_1 = f(\varepsilon_A)$

$\beta > |\beta_1|$

yes

no

$\beta = \beta_1$

$\beta = \beta \dfrac{\beta_1}{|\beta_1|}$

$\alpha = \alpha_1$
With Limits
$\alpha \leqq \pm \sqrt{(30)^2 - \beta_1^2}$

α, β

$\alpha = \alpha_1$
With Limits
$(\alpha_2 + b) \geqq \alpha \geqq (\alpha_2 - c)$

Figure 4.- Control logic for proportioning angle of attack and roll
angle as a function of lateral and longitudinal range errors.

(a) Range-to-go as a function of α_2, showing the variation in α_2 allowable for longitudinal range control.
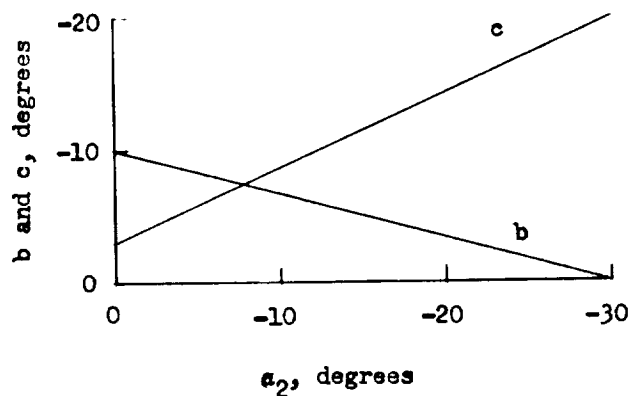
(b) Variation in b and c with α_2.

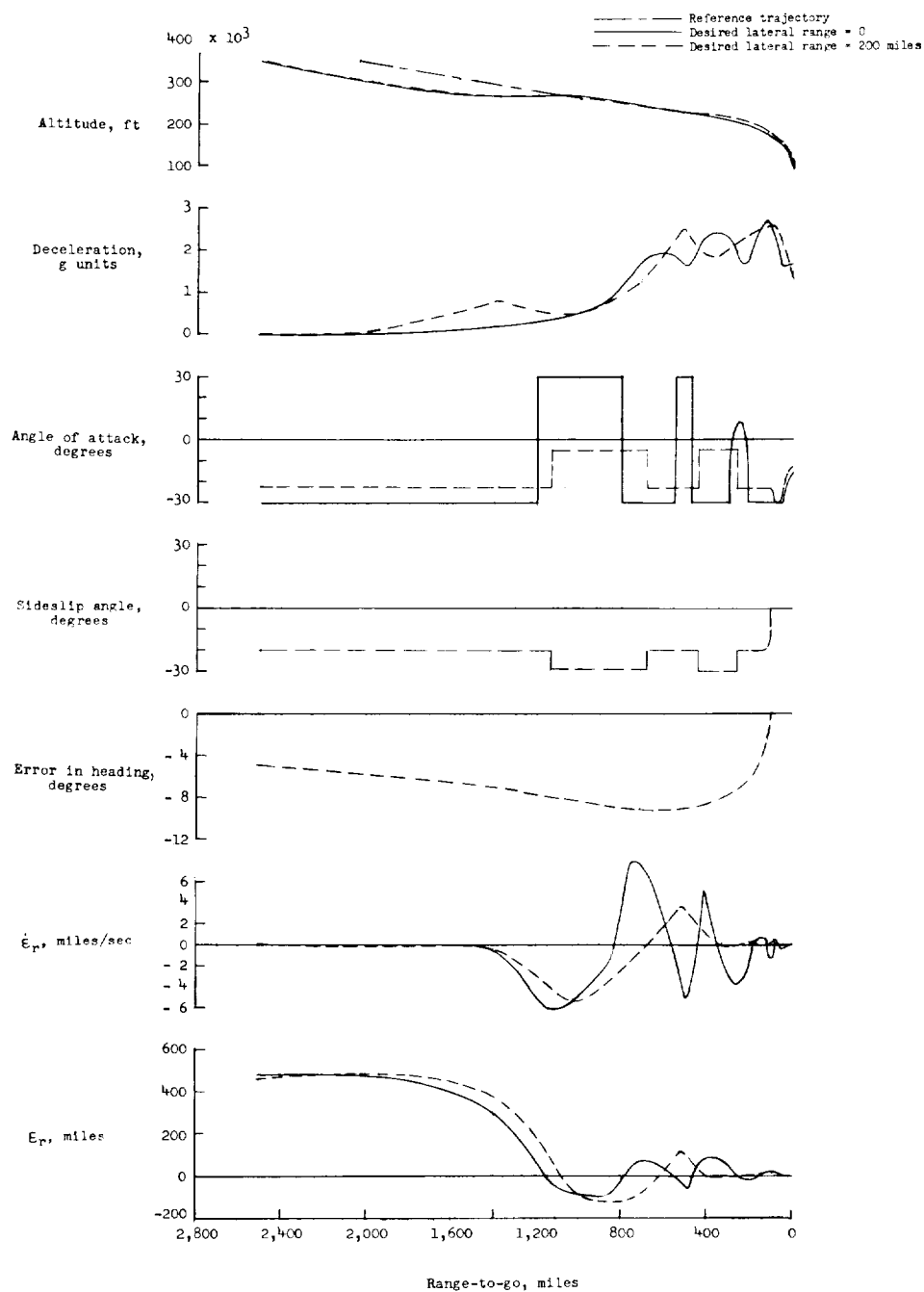Figure 5.- Parameters used in control logic.

Figure 6.- Variation in trajectory variables with range-to-go for equatorial entries toward the east with and without an initial heading error. h_0 = 350,000 feet; $V_{i,0}$ = 25,710 feet per second; γ_0 = -1°; W/S = 20 pounds per square foot.
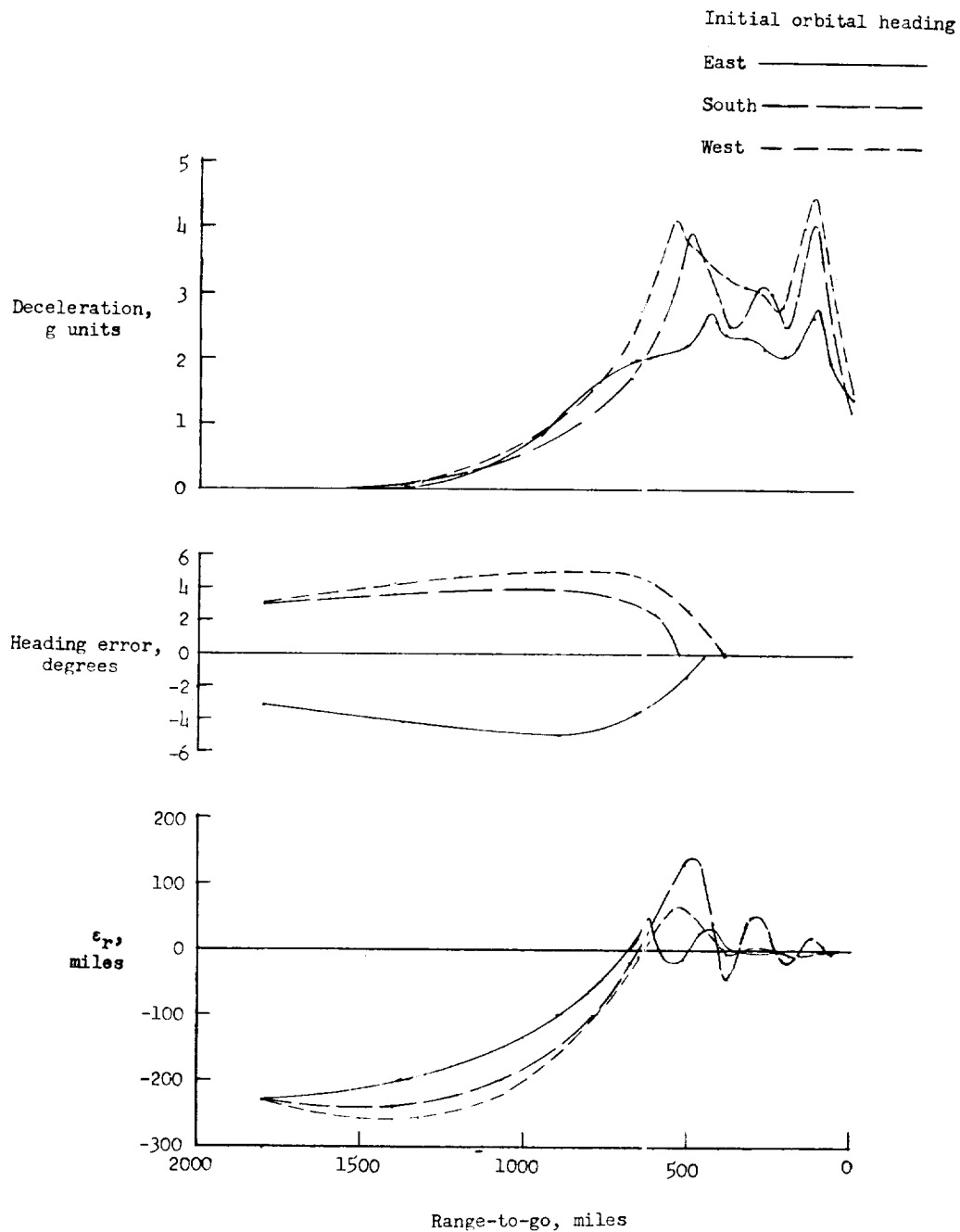
Figure 7.- Variation in range-to-go error, heading error, and deceleration for entries toward the east, south, and west with a desired longitudinal range of 1,800 miles and a desired lateral range of 100 miles. h_o = 350,000 feet; $V_{i,o}$ = 25,710 feet per second; γ_o = -1°; W/S = 20 pounds per square foot.
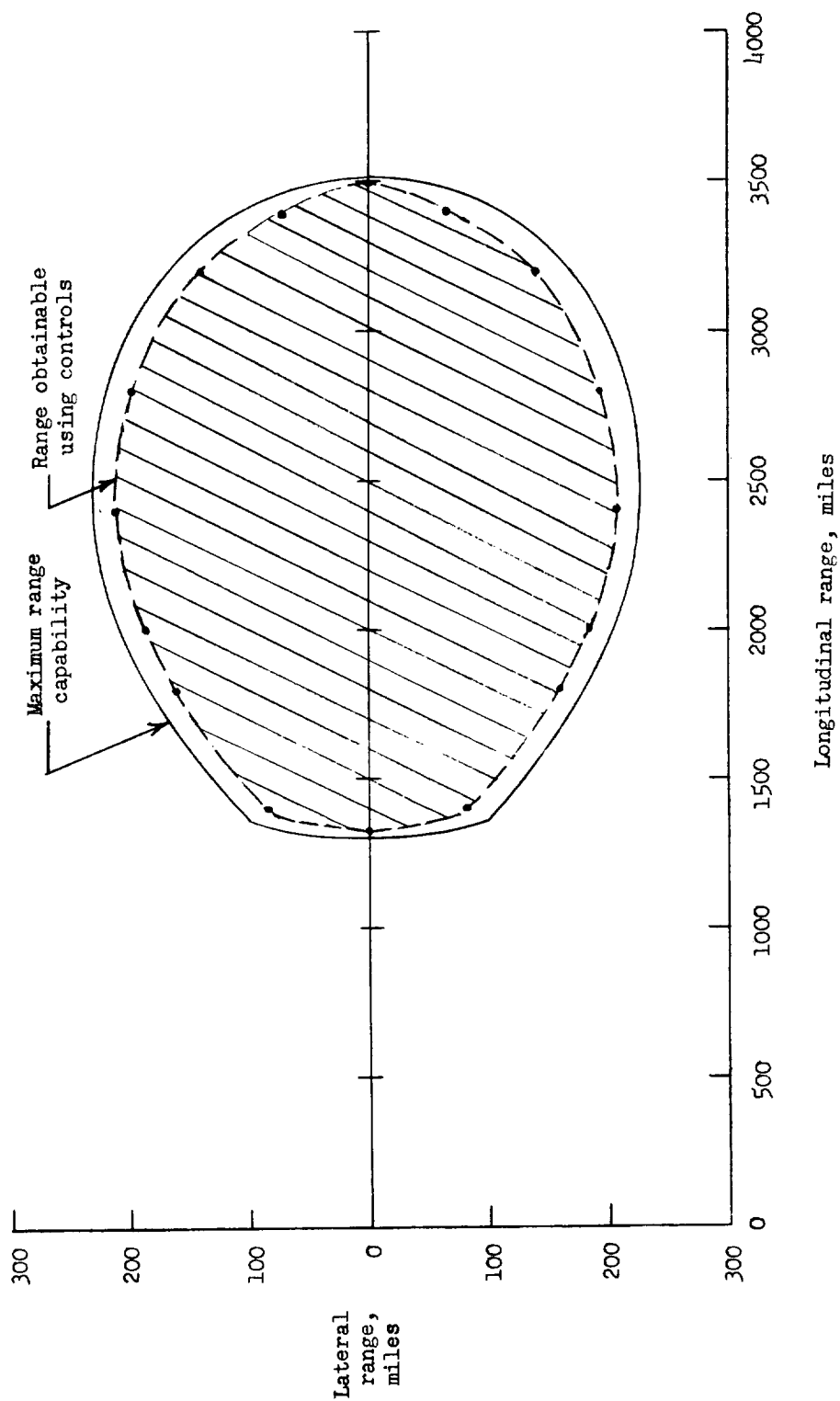
Figure 8.- Locus of end points of trajectories for equatorial entries toward the east showing maximum range capability and area in which vehicle can be controlled to within 10 miles of desired landing area. $h_o = 350,000$ feet; $V_{i,o} = 25,710$ feet per second; $\gamma_o = -10°$; $W/S = 20$ pounds per square foot.

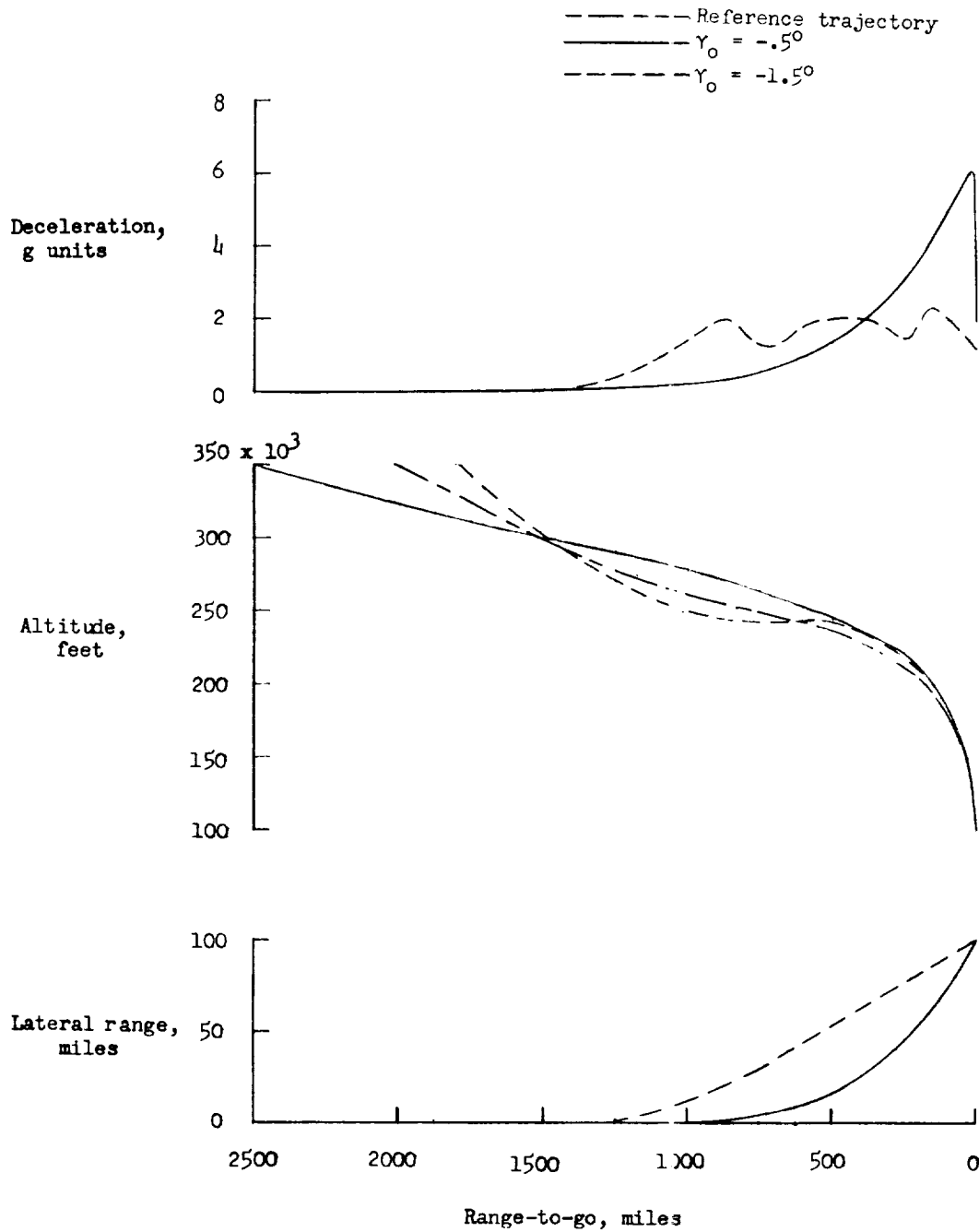Figure 9.- Variation in altitude, deceleration, and lateral range with range-to-go entries toward the east with initial flight-path angles of -0.5° and -1.5°. h_o = 350,000 feet; $V_{1,c}$ = 25,710 feet per second; W/S = 20 pounds per square foot.